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# Optimization of Electric Propulsion Flights between Periodic Orbits around Libration Points in the Earth-Moon System

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**Abstract**

Currently, many countries are developing concepts for lunar exploration. These concepts require the availability of servicing spacecraft, which should best perform communications, telemetry, reconnaissance, and retransmission tasks. At a result, periodic orbits such as Lyapunov orbits, halo orbits, axial orbits, vertical orbits, etc, which exist in the circular restricted three-body problem (CRTBP), are potential working orbits of interest for servicing spacecraft locations. The implementation of transfer between such orbits makes it possible to reduce the workload of libration point orbits and ensure the movement of spacecraft to various regions of the cislunar space. Compared to chemical engines, low-thrust electric rocket engines have the advantage of high specific impulse, high efficiency, and long operating time, which makes them suitable for long-term missions. However, optimizing the nominal control for low-thrust transfers is the most difficult part of the mission design. The study of low-thrust transfers in the CRTBP has been carried out by a small number of authors, who pointed out the poor convergence and low computational efficiency of optimization algorithms due to the strong nonlinearity of the mathematical model of motion. Therefore, it is desirable and important to develop algorithms for optimizing nominal control of low-thrust spacecraft orbital transfers in the CRTBP. This study aims to develop algorithms for determining the optimal nominal control of an electric propulsion spacecraft between periodic orbits around the libration points L1 and L2 in the Earth-Moon CRTBP. The methodology is based on the Pontryagin maximum principle, for three optimality criteria: the minimum flight time, the minimum energy costs and the minimum fuel consumption using continuation and homotopy methods. The proposed algorithm can be used for fast and efficient computation of optimal nominal control laws and corresponding trajectories between periodic orbits in the Earth-Moon system without the difficulty of finding an initial approximate solution. The calculated optimal control laws and transfer trajectories between periodic orbits in the Earth-Moon system can be used as references for the ballistic design of real missions.
**Keywords:** Optimal control; Transfer trajectory; Electric rocket engines; Pontryagin maximum principle; the Earth-Moon CRTBP

## 1. Introduction

The concept of the Lunar Orbital Platform-Gateway (LOP-G) is currently being developed by many countries. LOP-G can be used as a platform for deep space exploration of the Moon and other planets [1]. The creation of LOP-G requires servicing spacecraft that should best perform the tasks of communication, reconnaissance and navigation. At a result, periodic orbits such as Lyapunov orbits, halo orbits, axial orbits, vertical orbits, etc, which exist in the circular restricted three-body problem (CRTBP) are potential operational orbits of interest for servicing spacecraft [2].

The implementation of transfer between such orbits makes it possible to reduce the workload of libration point orbits (LPO) and ensure the movement of spacecraft to various regions of the cislunar space. One way to improve the effectiveness of research missions is to use the strategy of spacecraft flights with low-thrust engines [3]. The interest in lunar exploration raises the challenge of maneuvering between periodic orbits in the Earth-Moon system [4-8].

Low-thrust electric rocket engines offer the advantages of high specific impulse, high efficiency, and long operating time compared to chemical engines, making them suitable for long-term missions [9]. Researchers in the field of low-thrust transfer optimization in the CRTBP note the poor convergence of traditional optimization algorithms and low computational efficiency [10-12]. Classical shooting iterative methods require to determine a reasonably good initial approximation.

In this paper, the collocation method [13] is used as the iterative method, and a set of algorithms are proposed to select initial approximations that allow using the continuation methods to gradually shift from some known results to the final transfer trajectories. These algorithms are simple and have a high probability of success, making it useful to solve orbital transfer problems at the planning stage of future lunar missions.

## 2. The equations of spacecraft motion and the statement of the optimal control problem

The dimensionless equations of spacecraft controlled motion with a low-thrust engine in the Earth-Moon system can be written in the following vector form, taking into account the fuel consumption and engine state in the rotational coordinate system:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \boldsymbol{\alpha}, u) \Rightarrow \begin{bmatrix} \dot{\mathbf{r}} \\ \dot{\mathbf{v}} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \\ \partial U / \partial x + 2\dot{y} + C_1 u T_{\max} \alpha_x / m \\ \partial U / \partial y - 2\dot{x} + C_1 u T_{\max} \alpha_y / m \\ \partial U / \partial z + C_1 u T_{\max} \alpha_z / m \\ -C_2 u T_{\max} / c \end{bmatrix}, \tag{1}$$

where $U = \frac{1}{2}\left(x^2 + y^2\right) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}$ - the pseudopotential of the system; $r_1 = \sqrt{(x+\mu)^2 + y^2 + z^2}$ is the distance from the spacecraft to the Earth, while $r_2 = \sqrt{(x+\mu-1)^2 + y^2 + z^2}$ represents the distance from the spacecraft to the Moon; $\mu = 0.01215$ is the mass ratio of the Moon to the Earth-Moon system; $\mathbf{r} = [x, y, z]^{\mathrm{T}}$ и $\mathbf{v} = [v_x, v_y, v_z]^{\mathrm{T}}$ - vectors defining the position and velocity of the spacecraft; $m$ – current spacecraft mass; $T_{\max}$ - maximum thrust; $c = I_{sp} g_0$ - flow velocity of the propellant ( $I_{sp}$ - specific engine impulse; $g_0$ - the standard acceleration of gravity at sea level); The throttle thrust factor $u \in [0,1]$ and unit vector of thrust direction $\boldsymbol{\alpha}$ are the control variables; $C_1 = \tau_*^2 / l_*$ - constant that provides dimensionless acceleration from the engine thrust; $C_2 = \tau_*$ - the time scale constant for the propellant flow.

The unit thrust direction vector $\boldsymbol{\alpha}$ is determined by:

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{bmatrix} = \begin{bmatrix} \cos\theta_1 \cos\theta_2 \\ \sin\theta_1 \cos\theta_2 \\ \sin\theta_2 \end{bmatrix}, \tag{2}$$

where $\theta_1$ – control angle between the *Ox* axis and the projection of the thrust on the *xOy* plane, a $\theta_2$ – the angle between the thrust vector and the *xOy* plane.

Table 1 lists the physical parameters used in this work.

Table 1 List of parameters

| Parameters | Values |
|---|---|
| $I_{sp}$ | 2000 s |
| $l_*$ | 384400 km |
| $s_*$ | 1024,5 m / s |
| $\tau_*$ | 375675,84 s |
| $m$ | 1500 kg |

Three interrelated optimal control formation problems with different criteria are considered: min-time $J_t = \int_{t_0}^{t_f} 1\, dt$ , min-energy $J_e = \frac{T_{\max}}{c}\int_{t_0}^{t_f} u^2\, dt$ and min-fuel $J_f = \frac{T_{\max}}{c}\int_{t_0}^{t_f} u\, dt$ . Given the introduction of the homotopic parameter ε , the problems of optimal energy and optimal fuel consumption are related by the formula [9]:

$$J_{ef} = \frac{T_{\max}}{I_{sp} g_0}\int_0^{t_f} \left[u - \varepsilon u(1-u)\right] dt, \ \ \varepsilon \in [0,1]. \tag{3}$$

$$H = \boldsymbol{\lambda}_r^{\mathrm{T}} \mathbf{v} + \boldsymbol{\lambda}_v^{\mathrm{T}} \left( \mathbf{g}(\mathbf{r}) + \mathbf{h}(\mathbf{v}) + \frac{T_{\max} u C_1}{m} \boldsymbol{\alpha} \right) - \lambda_m \frac{T_{\max} u C_2}{c} + \begin{cases} 1, & \text{min-time}, \\ \dfrac{T_{\max} C_2}{c} \left( u - \varepsilon u (1-u) \right), & \text{min-energy and min-fuel}. \end{cases} \tag{4}$$

Applying the Pontryagin maximum principle, the optimal direction of the thrust vector is determined by $\boldsymbol{\alpha}^* = -\frac{\boldsymbol{\lambda}_v}{\|\boldsymbol{\lambda}_v\|}$. The control angles $\theta_1$ and $\theta_2$ are written as follows:

$$\tan\theta_1 = \frac{-\lambda_{v_y}}{-\lambda_{v_x}}, \qquad \tan\theta_2 = \frac{-\lambda_{v_z}}{-\lambda_{v_x}} \cdot \cos(\theta_1). \tag{5}$$

For the problem of min-time transfer, the engine runs without shutdowns. For min-energy and min-fuel transfer problem, the thrust throttling function $u^*$ is defined by the expression

$$u^* = \begin{cases} 0, & 1 - \dfrac{C_1 \|\boldsymbol{\lambda}_v\| c}{C_2 m} - \lambda_m > \varepsilon, \\ \left( \varepsilon - 1 + \dfrac{C_1 \|\boldsymbol{\lambda}_v\| c}{C_2 m} + \lambda_m \right) \Big/ 2\varepsilon, & -\varepsilon \le 1 - \dfrac{C_1 \|\boldsymbol{\lambda}_v\| c}{C_2 m} - \lambda_m \le \varepsilon, \\ 1, & 1 - \dfrac{C_1 \|\boldsymbol{\lambda}_v\| c}{C_2 m} - \lambda_m < -\varepsilon. \end{cases} \tag{6}$$

Thus, the optimal control problem is reduced to a 6-parameter boundary value problem: the boundary and transversality conditions must be satisfied for the two starting and ending points $\mathrm{M}(\mathbf{r}_0, \mathbf{v}_0)$ and $\mathrm{N}(\mathbf{r}_f, \mathbf{v}_f)$:

$$\begin{aligned} &\text{initial:} \quad \mathbf{r}(t_0) = \mathbf{r}_0,\ \mathbf{v}(t_0) = \mathbf{v}_0,\ m(t_0) = m_0, \\ &\text{final:} \quad \mathbf{r}(t_f) = \mathbf{r}_f,\ \mathbf{v}(t_f) = \mathbf{v}_f, \lambda_m(t_f) = 0. \end{aligned} \tag{7}$$

**3. Solution methods and computational techniques.**

Most of the methods proposed in current studies for calculating optimal transfer trajectories in the CRTBP are poorly convergent, computationally inefficient, and do not use all the features of gravity of the three-body system to reduce the requirements to the characteristics of the real engine product, such as the required thrust, engine operating time, and fuel consumption. In this paper, the following computational methods and solution technique are used to solve the problem of optimal transfer between periodic orbits in the Earth-Moon CRTBP without difficulty in finding an initial solution approximation.

*a) Collocation method*

The collocation method is used to integrate and update the initial approximations of the generated orbits. It provides an alternative method for generating orbits that is more reliable than the differential correction scheme, even for extremely poor initial guesses. The collocation method, as opposed to the Newton method, determines the values of the state vector and the costate vector at each sampling point.

*b) Parameter Continuation Method*

A transfer trajectory satisfying a specific condition is used as an initial approximation of the solution, and the parameter continuation method is then used to progress to the final solution. One of the most efficient methods for solving a series of complex boundary value problems is the parametric continuation method. Typically, one continuous parameter is used, and the problem has a known solution at one of its values. This method allows a transition from a known solution to a new one if the parameters are changed.

**4. Transfer simulation results**

Let us consider an example of an optimal transfer between Lyapunov orbits in the Earth-Moon system. It is clear that the flight time in the min-energy and min-fuel problems should be longer than when solving the optimization problem by the minimum flight time criterion.

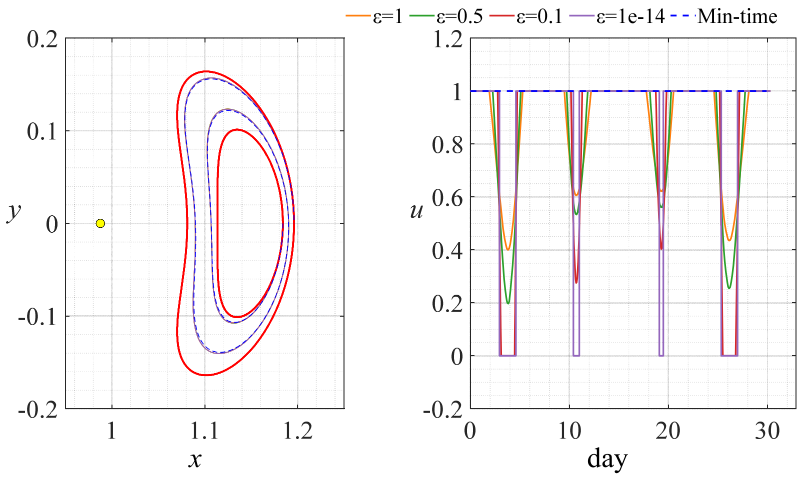


Fig. 1. Optimal transfer trajectories between L2 Lyapunov orbits with min-time, min-energy and min-fuel

As the homotopy parameter decreases, the problem would gradually shift from $\varepsilon=1$ (min-energy) to $\varepsilon=0$ (min-fuel). However, the shapes of the trajectories corresponding to different parameters do not differ much from each other.

The calculations show that with a gradual increase in the flight time $t_f$ from the min-time $t_{f_{\min}}$, the relative final mass (the final mass of the spacecraft after the transfer divided by the starting mass) first increases and then decreases (Fig. 2a), i.e., there is a best flight time for the min-fuel transfer. The engine propellant decreases from 10.5560 kg (min-time) to 9.1741 kg.

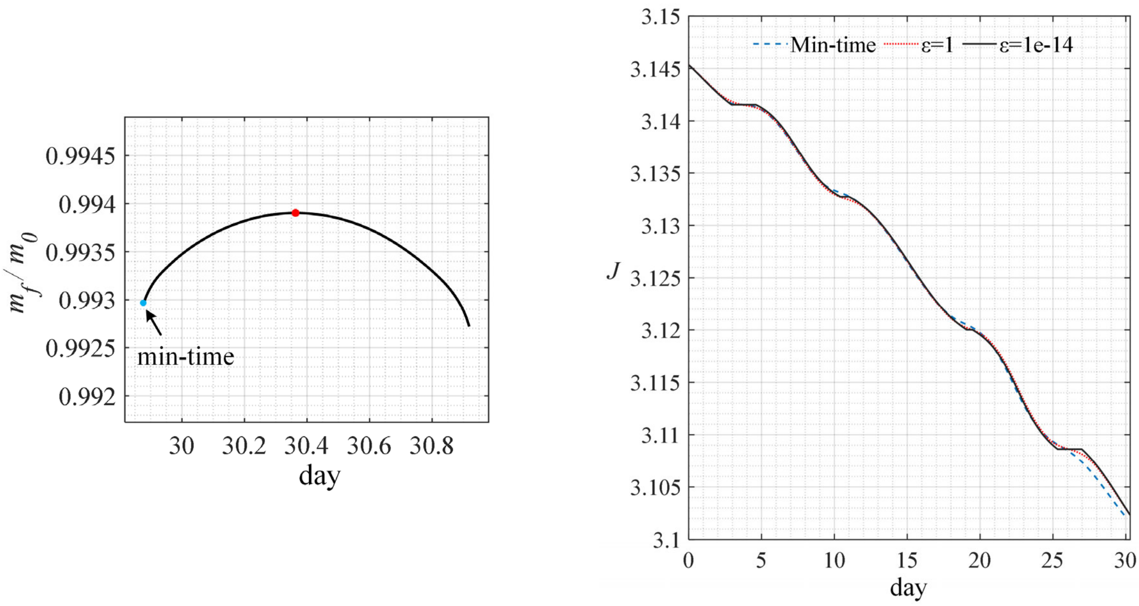


Fig.2. a) Change in the relative mass of the spacecraft, b) Change in the Jacobi constant as a function of flight time

Comparing the behavior of the Jacobi constant along the flight paths plotted in Fig. 1, it can be seen that with the control law ensuring a min-fuel transfer, the Jacobi constant J is a step monotonic function along the optimum flight path, as shown in Fig. 2b. When the engine is switched off (u=0), the Jacobi constant is naturally unchanged.

It should be noted that there are many solutions to the boundary value problem, varying in duration and angular extent. For example, Fig. 3 shows the results of optimal control for the transfer problem between periodic orbits around the libration points L2 and L1, using invariant manifolds with different numbers of turns relative to the Moon as initial trials.

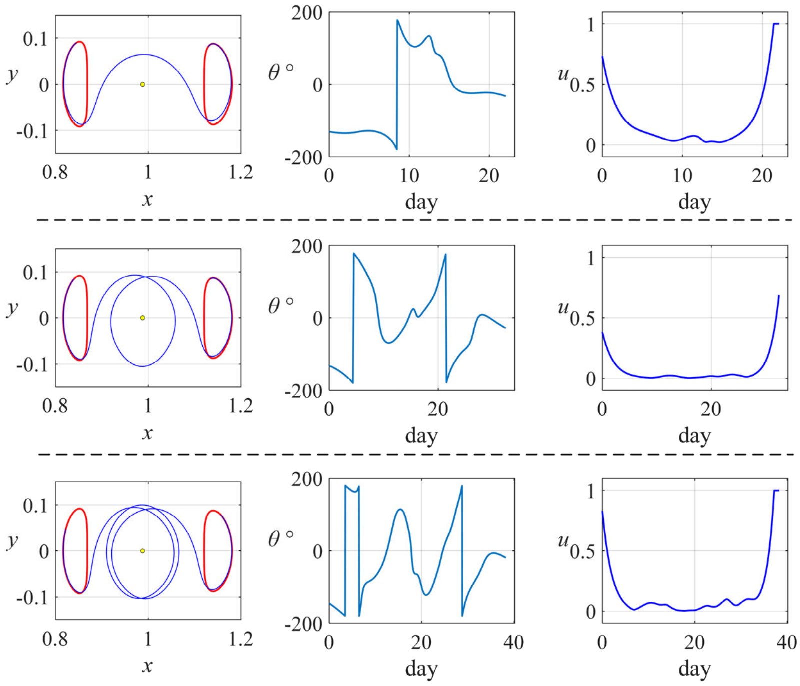


Fig.3. Low-thrust energy-optimal heteroclinic transfers from L2 Lyapunov orbit to L1 Lyapunov orbit: (top) half-rev around the Moon; (middle) one-rev; (bottom) two-rev

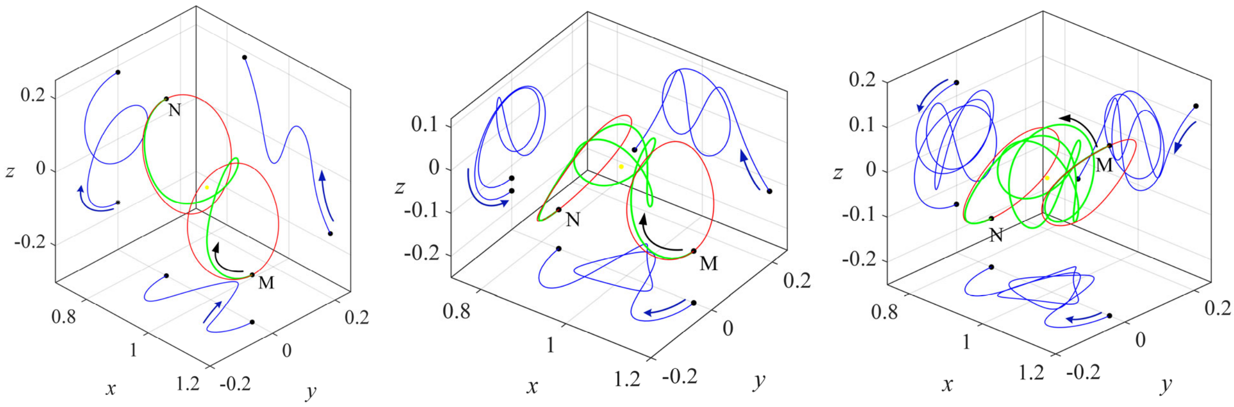


Fig.4. Low-thrust time-optimal transfers from L2 halo orbits to L1 halo orbits: (top) half-rev around the Moon; (middle) one-rev; (bottom) two-rev

**5. Conclusions**

In this paper, we develop computational methods for the formation of optimal nominal controls for spacecraft with low-thrust engines to transfer between periodic orbits in the Earth-Moon system according to the criteria of -time, -energy, and -fuel, including the construction of initial approximations for iterative processes for the solution of boundary value problems. A continuous algorithm combined with a collocation method is proposed to solve the optimization problem. Simulation results confirm the effectiveness of this computational procedure.

The results allow us to state that the developed methods and algorithms allow us to obtain solutions to the problem of optimal transfer between periodic orbits in the Earth-Moon system.

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